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## Probing Large Distance Higher-Dimensional Gravity with Cosmic Microwave Background Measurements

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### Abstract

It has been recently argued that higher dimensional gravity theories may manifest themselves not only at short microscopic distances but also at large cosmological scales. We study the constraints that cosmic microwave background measurements set on such large distance modifications of the gravitational potential.

The possibility that the Universe may be higher-dimensional with standard matter and radiation localized on a four-dimensional surface known as a (3-)brane has attracted a lot of attention recently. This obviously would have important cosmological as well as astrophysical consequences: only the tip of this iceberg has been explored.

In most of the models constructed until now, the laws of gravitation between two test masses on a 4-dimensional brane are standard 4-dimensional laws, for distances larger than a typical scale of order the radius of the compact dimensions [1]. When one probes shorter distances, the full dimensions open up and the laws of gravitation become higher-dimensional.

Some of these extra dimensions may not even be compact. In the so-called Randall-Sundrum scenario [2, 3], even though a fifth dimension is non-compact, there is a normalisable massless mode among the 5-dimensional metric fluctuation modes, which is interpreted as the standard 4-dimensional graviton. Strictly speaking, the fifth dimension is not infinite in the sense that there is an horizon at finite distance from the brane. Also, the bulk of spacetime has a simple anti-de Sitter structure.

The 4-dimensional matter feels long-distance gravity through its interaction with the 5-dimensional bulk. In the case where this bulk has not such a simple structure as in the Randall-Sundrum model, one may expect that the larger the distance is, the more sensitive one becomes to the structure of the bulk, to the other branes which it may contain and thus to higher dimensions. This could have the effect that, at very large (cosmological?) distances, one may recover higher-dimensional gravity or at least be sensitive to scalar exchange gravity, given the presence of moduli fields describing compact dimension radii, or distances between branes in the bulk.

For example, if the large-distance physics associated with the cosmological constant is to be stabilized through some short-distance cancellations ensured by some bulk supersymmetry [4], again one may expect that probing large distances on the brane may reveal violations of standard 4-dimensional gravitational laws.

Several models [5, 6] which have been proposed recently show the type of behaviour that we consider here.

In the GRS model [6], the (positive tension) 3-brane is located in the middle of a anti-de Sitter slab limited by two negative tension branes and flat Minkowski space on either side. The solution considered for the metric includes a warp factor of the Randall-Sundrum type. However, there are no normalisable zero mode. Instead, for an intermediate range of distances between two test masses on the 3-brane, the exchange of the collection of non-normalisable graviton modes mimics the exchange of a single 4-dimensional

graviton. This graviton is interpreted as a metastable state: when the distance becomes too large, it decays and one expects to recover 5-dimensional gravity.

There is however a debate over the question whether the GRS model is internally consistent [7]-[12]. The presence of a negative tension brane violates the weakest form of a positive energy condition [8]. Moreover [10], the exchange of the radion scalar field (whose vacuum expectation value fixes the interbrane distance –the radius–) generates scalar antigravity which dominates at very large distances. This antigravity is clearly associated with the presence of negative tension branes. It remains to be seen whether it is unavoidable [9] and if the model remains inconsistent at the quantum level. This source of instability might also be cancelled by other scalar field exchanges, in which case one would recover at large distances the 5-dimensional gravity behavior.

In the models of Kogan *et al.* [5], the dimensions are compact and exotic large distance effects are due to the presence of very light Kaluza-Klein states. It was however argued [11] that this type of model and the preceding one belong to the same general class. These models also have the possible drawbacks associated with negative tension branes.

Finally, it has been stressed recently [13] that, when one tries to give a small mass to localized scalars, the zero modes turn into quasi-localized states with finite decay width: their exchange generates a potential with a power law behavior at large distances.

In what follows, we will not rest on any specific model but consider the case where gravity becomes five-dimensional at very large distances, only commenting briefly on other possibilities. How large must such distances be? Milgrom [14] has extensively discussed deviations from Einstein gravity on galactic scales in order to account for galaxy rotation curves. However because dark matter is probed in virialised systems over a wide range of density on scales from kpc to tens of Mpc, it is clear that a simple change in the force law, such as we consider here, could only occur on substantially larger scales. Hence if such violations appear at any macroscopic distances other than cosmological, existing limits severely constrain them. In the case of violations at cosmological distances, we argue in this note that cosmological background and deep galaxy redshift survey measurements may provide useful limits or interesting ways of probing such theories.

We recall that in standard gravity, curvature or metric fluctuations scale as

$$\delta\varphi = \frac{G\delta M}{rc^2} = \left(\frac{\delta M}{M}\right) \left(\frac{r}{ct}\right)^2.$$

On subhorizon scales, the linear mass fluctuations in a gaussian-distributed density field [15] are

$$\frac{\delta M}{M} \approx 0.1 \left( \frac{M_{eq}}{M} \right)^{\frac{n+3}{6}} \quad (M > M_{eq}) \quad (1)$$

$$\approx \text{constant } M^{\frac{n-1}{6}} \quad (M \ll M_{eq}) \quad (2)$$

where  $M_{eq}$  is the horizon scale at the matter-radiation equal density epoch and the density fluctuation power spectrum has been taken to be  $P(k) \propto k^n$ . CMB measurements confirm approximately scale-invariant ( $n \approx 1$ ) fluctuations  $n = 1.2 \pm 0.3$  [16] and the power spectrum normalization on scales near  $M_{eq}$  may be deduced from the height of the first acoustic peak [17], to within an uncertainty of at most a factor of 2, corresponding to the bias between mass fluctuations (measured by the CMB, but dependent on cosmological model parameters) and the fluctuations in the luminous mass density (inferred from large-scale structure surveys) [18].

If the gravity force changes on scale  $r_s$  to a 5-dimensional law, the metric fluctuations can be expanded as

$$\delta\phi = \left( \frac{\delta M}{M} \right) \left( \frac{r}{ct} \right)^2 \left( \frac{r_s}{r + r_s} \right).$$

Hence a scale-invariant fluctuation spectrum, as predicted by most inflation and defect models for the fluctuations, results in large-scale power with variance  $\delta M/M \propto M^{-1/3}$  as opposed to the  $M^{-2/3}$  predicted for the standard gravity model on scales  $r \ll r_s$ . This would not be visible as primary fluctuations on the last scattering surface of the CMB if  $r_s$  is of order the horizon scale, since the Sachs-Wolfe effect is generated by the constant potential fluctuations on horizon scales. However it should give a signal that is potentially measureable in deep redshift surveys such as 2DF and SDSS, which may eventually probe to 500 Mpc with galaxies, and can, using quasars, potentially probe much large scales.

However the CMB fluctuations do provide a possible constraint on the scale of higher dimensional gravity via the integrated Sachs-Wolfe effect. This measures  $\int \delta\dot{\phi} c dt$  since last scattering. The linear fluctuation growth rate is modified above scale  $r_s$ . To demonstrate this, consider the Newtonian limit for the perturbation equations as a simple approximation valid in the matter-dominated era on subhorizon scales. One has

$$\left( \frac{\partial^2}{\partial t^2} + \frac{2\dot{a}}{a} \frac{\partial}{\partial t} \right) \delta_k + \frac{k^2}{a^2} \left( \frac{dp}{d\rho} - \varphi_k \right) \delta_k = 0,$$

where  $a(t)$  is the cosmological scale factor and  $\rho(\propto a^{-3})$  is the density of non-relativistic matter. 5-dimensional gravity in the dust limit modifies the potential in the small-scale limit to

$$a^{-2}k^2\varphi_k^{(5)} = 4\pi G\rho(k/k_s), \quad (3)$$

and a power-law solution  $\delta_k \propto t^n$  satisfies

$$n^2 + n/3 - (2/3)(k/k_s) = 0.$$

Hence growth is suppressed on scales  $r > r_s$ , and the usual Jeans length is modified to

$$\frac{k_J}{a} = \frac{4\pi G\rho}{dp/d\rho} \frac{a}{k_s} \quad \text{or} \quad L_J = \frac{2\pi a}{k_J} = r_s^{-1}v_s^2/(2G\rho),$$

where  $v_s^2 = dp/d\rho$ . The peak in the power spectrum of primordial fluctuations is at  $r_{eq}$ , but the shape is slightly modified on larger scales

$$r_{eq} \lesssim r \lesssim r_{h,ls} \quad (4)$$

because of the suppression in subhorizon growth prior to last scattering. On scales larger than the last scattering horizon  $r_{h,ls}$ , only the potential fluctuations contribute to  $\delta T/T$ . The change in growth rate affects the shape of  $P(k)$  near the peak, where power will be suppressed. The contribution of the integrated Sachs-Wolfe effect is also reduced.

More detailed numerical calculations are needed to produce precise numbers, but it seems clear that the current precision of CMB measurements on the scale range where growth modification is most important already constrains  $r_s \gtrsim r_{h,ls}$ . These effects will not be easy to disentangle from the data because of the effects of cosmic variance, but a combination of redshift survey and CMB data should be able to set significant limits on  $r_s$ .

One may also wonder whether the range of scale (4) may be selected from the point of view of the fundamental theory. Let us note here only that such scales are within a few orders of magnitude of the length scale associated with the cosmological constant [19] and may therefore be associated with another cosmic coincidence [20]. Indeed, the type of theories that we consider here have been advocated for a partial solution of the cosmological constant problem.

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